European Shout Cap and Floor

A European shout cap is an option giving the holder the right to “shout” a European call strike level at spot at any time during the option tenor. That is, the holder receives an at-the-money European call when they shout. If they do not shout at any time during the option tenor the holder receives a European call struck at the initial strike level. Typically the initial strike level is set to the spot level when the contract is initiated. This instrument provides a protective cap on losses in short positions without requiring additional payments.

A European shout floor is similar to a shout cap. The shout floor is an option giving the holder the right to “shout” a European put strike level at spot at any time during the option tenor. That is, the holder receives an at-the-money European put when they shout. If they do not shout at any time during the option tenor the holder receives a European put struck at the initial strike level. This instrument provides a protective floor on losses in long positions without requiring additional payments (see https://finpricing.com/lib/FxCompound.html).

It should be noted that shout caps and floors may be referred to as shout calls and puts although shout calls and puts originally referred to differently structured shout options

Let $S(t)$ denote the underlying asset value at time t . Under the risk neutral measure it is assumed that $S(t)$ follows geometric Brownian motion with deterministic and time dependent drift and volatility. $S(t)$ may refer to an equity or a foreign exchange rate.

At expiry the payoffs for shout caps and floors are given as follows:

$$X(\tau)=\begin{cases}\max(S(T)-S(\tau),0), & 0\le\tau<T\\ \max(S(T)-K,0), & otherwise\end{cases}\qquad \text{for a cap}$$

$$X(\tau)=\begin{cases}\max(S(\tau)-S(T),0), & 0\le\tau<T\\ \max(K-S(T),0), & otherwise\end{cases}\qquad \text{for a floor}$$

where

· K is the initial strike level,

· T is the option tenor, and

· t is the time when the strike level is shouted

It should be noted that t is stochastic. For example consider how a shout floor holder decides to shout the strike level. At any time t where 0 £ t £ T , if the state of the world is such that value of shouting (receiving an at-the-money European put) is less than the value of not shouting then t is greater than t . For a different state of the world we may have t =t . Thus for any information set at time t the holder can decide whether to shout (t =t) or not (t <t). Thus t is stochastic. (Strictly speaking, t is a stopping time).

The price of European shout caps and floors are given by choosing a shouting strategies that maximize the present value of the expectation of their respective payoffs at expiry. The optimal shouting strategy amounts to specifying a stopping time which is knowing whether to shout or not at a given time in a given state of the world that takes values in the interval [0,T] since the option tenor is T . If t denotes the optimal shout strategy for a European shout option then the shout option price is given by

$$V = E_0\left[B^{-1}(T)X(\tau)\right] \qquad (1)$$

where

· $B(T)$ is the money market account which is the value of continuously reinvesting one dollar over the option tenor, and

· X(t) is the terminal value for shout cap or floor as specified above.

It should be noted that optimal shouting strategies for a shout cap or floor need not be the same.

The pricing of European cap and shout floors are optimal stopping problems for which analytic solutions do not exist unless $K = 0$. For K ¹ 0, they must be solve numerically. The prices are determined numerically using tree based methods. The option tenor is discretized and for each time in the discretization a discrete set of states for the underlying asset is constructed [2]. Given the tree of state prices, the shout cap and floor prices are obtained by backward inducting through the tree from expiration comparing the value of shouting to the shout option value and taking the maximum value.

The backward induction for a shout cap specified as follows:

$$V(t) = \max(Call, \exp(-r\Delta t)E_t\left[V(t+\Delta t)\right]) \qquad (3)$$

· *Call* is the intrinsic value of shouting which is a European call struck at spot with tenor T - t ;

· exp(-*r* *t*)*E* [*V*(*t* + *t*)] *t* D D is the time *t* present value of the expected value of the shout cap value at time *t* + D*t* .

· the terminal condition is *V*(*T*) = max(*S*(*T*) - *K*,0) .

The backward induction for as shout put specified as follows:

$$V(t) = \max(Put, \exp(-r\Delta t)E_t[V(t+\Delta t)]) \quad (4)$$

with terminal condition *V*(*T*) = max(*K* - *S*(*T*),0)

· *Put* is the intrinsic value of shouting which is a European put struck at spot with tenor *T* - *t* ;

· exp(-*r* *t*)*E* [*V*(*t* + *t*)] *t* D D is the time *t* present value of the expected value of the shout cap value at time *t* + D*t* .

· the terminal condition is *V*(*T*) = max(*K* - *S*(*T*),0) .